

Pointwise convergence in probability of general smoothing splines

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Abstract Establishing the convergence of splines can be cast as a variational problem which is amenable to a Γ -convergence approach. We consider the case in which the regularization coefficient scales with the number of observations, n , as $\lambda_n = n^{-p}$. Using standard theorems from the Γ -convergence literature, we prove that the general spline model is consistent in that estimators converge in a sense slightly weaker than weak convergence in probability for $p \leq \frac{1}{2}$. Without further assumptions, we show this rate is sharp. This differs from rates for strong convergence using Hilbert scales where one can often choose $p > \frac{1}{2}$.

Keywords Variational methods · Γ -convergence · Pointwise convergence · General spline model · Nonparametric smoothing

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