

# Fold-up derivatives of set-valued functions and the change-set problem: A Survey

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**Abstract** We give a survey on fold-up derivatives, a notion which was introduced by Khmaladze (J Math Anal Appl 334:1055–1072, 2007) and extended by Khmaladze and Weil (J Math Anal Appl 413:291–310, 2014) to describe infinitesimal changes in a set-valued function. We summarize the geometric background and discuss in detail applications in statistics, in particular to the change-set problem of spatial statistics, and show how the notion of fold-up derivatives leads to the theory of testing statistical hypotheses about the change-set. We formulate Poisson limit theorems for the log-likelihood ratio in two versions of this problem and present also the route to a central limit theorem.

**Keywords** Infinitesimal image analysis · Generalized functions · Fold-up derivatives · Local Steiner formula · Local point process · Set-valued mapping · Derivative set · Normal cylinder · Change-set problem

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