

On coupon collector's and Dixie cup problems under fixed and random sample size sampling schemes

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Abstract Suppose an urn contains m distinct coupons, labeled from 1 to m . A random sample of k coupons is drawn without replacement from the urn, numbers are recorded and the coupons are then returned to the urn. This procedure is done repeatedly and the sample sizes are independently identically distributed. Let W be the total number of random samples needed to see all coupons at least l times ($l \geq 1$). Recently, for $l = 1$, the approximation for the first moment of the random variable W has been studied under random sample size sampling scheme by Sellke (Ann Appl Probab, 5:294–309, 1995). In this manuscript, we focus on studying the exact distributions of waiting times W for both fixed and random sample size sampling schemes given $l \geq 1$. The results are further extended to a combination of fixed and random sample size sampling procedures.

Keywords Coupon collector's problems · Dixie cup problems · Finite Markov chain imbedding

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