

On the coverage probabilities of parametric confidence bands for continuous distribution and quantile functions constructed via confidence regions for a location-scale parameter

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Abstract In parametric statistics, confidence bands for continuous distribution (quantile) functions may be constructed by unifying the graphs of all distribution (quantile) functions corresponding to parameters lying in some confidence region. It is then desirable that the coverage probabilities of both, band and region, coincide, e.g., to prevent from wide and less informative bands or to transfer the property of unbiasedness; this is ensured if the confidence region is exhaustive. Properties and representations of exhaustive confidence regions are presented. In location-scale families, the property of some confidence region to be exhaustive depends on the boundedness of the supports of the distributions in the family. For unbounded, one-sided bounded and bounded supports, characterizations of exhaustive confidence regions are derived. The results are useful to decide whether the trapezoidal confidence regions based on the standard pivotal quantities are exhaustive and may serve to construct exhaustive confidence regions in (log-)location-scale models.

Keywords Comprehensive convex hull · Confidence band · Confidence region · Coverage probability · Location-scale family · Simultaneous confidence intervals

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