

The uniqueness of the Fisher metric as information metric

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Received: 12 July 2013 / Revised: 4 April 2016 / Published online: 14 May 2016
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Abstract We define a mixed topology on the fiber space $\cup_{\mu} \oplus^n L^n(\mu)$ over the space $\mathcal{M}(\Omega)$ of all finite non-negative measures μ on a separable metric space Ω provided with Borel σ -algebra. We define a notion of strong continuity of a covariant n -tensor field on $\mathcal{M}(\Omega)$. Under the assumption of strong continuity of an information metric, we prove the uniqueness of the Fisher metric as information metric on statistical models associated with Ω . Our proof realizes a suggestion due to Amari and Nagaoka to derive the uniqueness of the Fisher metric from the special case proved by Chentsov by using a special kind of limiting procedure. The obtained result extends the monotonicity characterization of the Fisher metric on statistical models associated with finite sample spaces and complement the uniqueness theorem by Ay–Jost–Lê–Schwachhöfer that characterizes the Fisher metric by its invariance under sufficient statistics.

Keywords Monotonicity of the Fisher metric · Chentsov’s theorem · Mixed topology

H.V.L. is partially supported by RVO: 67985840.

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