

## Efficient ANOVA for directional data

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**Abstract** In this paper, we tackle the ANOVA problem for directional data. We apply the invariance principle to construct locally and asymptotically most stringent rank-based tests. Our semi-parametric tests improve on the optimal parametric tests by being valid under the whole class of rotationally symmetric distributions. Moreover, they keep the optimality property of the latter under a given  $m$ -tuple of rotationally symmetric distributions. Asymptotic relative efficiencies are calculated and the finite-sample behavior of the proposed tests is investigated by means of a Monte Carlo simulation. We conclude by applying our findings to a real-data example involving geological data.

**Keywords** Directional statistics · Local asymptotic normality · Pseudo-FvML tests · Rank-based inference · ANOVA

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