

# Extreme sizes in Gibbs-type exchangeable random partitions

Shuhei Mano<sup>1,2</sup>

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**Abstract** Gibbs-type exchangeable random partition, which is a class of multiplicative measures on the set of positive integer partitions, appear in various contexts, including Bayesian statistics, random combinatorial structures, and stochastic models of diversity in various phenomena. Some distributional results on ordered sizes in the Gibbs partition are established by introducing associated partial Bell polynomials and analysis of the generating functions. The combinatorial approach is applied to derive explicit results on asymptotic behavior of the extreme sizes in the Gibbs partition. Especially, Ewens–Pitman partition, which is the sample from the Poisson–Dirichlet process and has been discussed from rather model-specific viewpoints, and a random partition which was recently introduced by Gnedin, are discussed in the details. As by-products, some formulas for the associated partial Bell polynomials are presented.

**Keywords** Random partition · Extremes · Analytic combinatorics · The Bell polynomials · Gibbs partitions · The Ewens–Pitman partition · Poisson–Dirichlet process

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✉ Shuhei Mano  
smano@ism.ac.jp

<sup>1</sup> The Institute of Statistical Mathematics, 10-3 Midori-cho, Tachikawa, Tokyo 190-8562, Japan

<sup>2</sup> CREST, Japan Science and Technology Agency, 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan