Ann Inst Stat Math (2016) 68:765–785
DOI 10.1007/s10463-015-0514-0

Convergence of empirical spectral distributions of large dimensional quaternion sample covariance matrices

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Received: 4 June 2014 / Revised: 27 December 2014 / Published online: 22 March 2015

Abstract In this paper, we establish the limit of empirical spectral distributions of quaternion sample covariance matrices. Motivated by Bai and Silverstein (Spectral analysis of large dimensional random matrices, Springer, New York, 2010) and Marčenko and Pastur (Matematicheskii Sb, 114:507–536, 1967), we can extend the results of the real or complex sample covariance matrix to the quaternion case. Suppose $\mathbf{X}_n = (x_{jk}^{(n)})_{p\times n}$ is a quaternion random matrix. For each n, the entries $\{x_{ij}^{(n)}\}$ are independent random quaternion variables with a common mean μ and variance $\sigma^2 > 0$. It is shown that the empirical spectral distribution of the quaternion sample covariance matrix $\mathbf{S}_n = n^{-1}\mathbf{X}_n\mathbf{X}_n^*$ converges to the Marčenko–Pastur law as $p \to \infty$, $n \to \infty$ and $p/n \to y \in (0, +\infty)$.

Keywords Empirical spectral distribution · LSD · Quaternion matrices · Sample covariance matrix

Z. D. Bai was partially supported by CNSF 11171057, the Fundamental Research Funds for the Central Universities, and PCSIRT; J. Hu was partially supported by a grant CNSF 11301063.

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