

# Optimal restricted quadratic estimator of integrated volatility

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**Abstract** Estimation of the integrated volatility is an important problem in high-frequency financial data analysis. In this study, we propose a quadratic unbiased estimator of the integrated volatility for stochastic volatility models with microstructure noise. The proposed estimator minimizes the finite sample variance in the class of quadratic estimators based on symmetric Toeplitz matrices. We show the proposed estimator has an asymptotic mixed normal distribution with optimal convergence rate  $n^{-1/4}$  and achieves the maximum likelihood estimator efficiency for constant volatility case. Simulation results show that our proposed estimator attains better finite sample efficiency than state-of-the-art methods. Finally, a real data analysis is conducted for illustration.

**Keywords** High-frequency data · Integrated volatility · Microstructure noise · Signal-to-noise ratio · Stochastic volatility model

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