

Second-order asymptotic comparison of the MLE and MCLE of a natural parameter for a truncated exponential family of distributions

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Abstract For a truncated exponential family of distributions with a natural parameter θ and a truncation parameter γ as a nuisance parameter, it is known that the maximum likelihood estimators (MLEs) $\hat{\theta}_{ML}^\gamma$ and $\hat{\theta}_{ML}$ of θ for known γ and unknown γ , respectively, and the maximum conditional likelihood estimator $\hat{\theta}_{MCL}$ of θ are asymptotically equivalent. In this paper, the stochastic expansions of $\hat{\theta}_{ML}^\gamma$, $\hat{\theta}_{ML}$ and $\hat{\theta}_{MCL}$ are derived, and their second-order asymptotic variances are obtained. The second-order asymptotic loss of a bias-adjusted MLE $\hat{\theta}_{ML}^*$ relative to $\hat{\theta}_{ML}^\gamma$ is also given, and $\hat{\theta}_{ML}^*$ and $\hat{\theta}_{MCL}$ are shown to be second-order asymptotically equivalent. Further, some examples are given.

Keywords Truncated exponential family · Natural parameter · Truncation parameter · Maximum likelihood estimator · Maximum conditional likelihood estimator · Stochastic expansion · Asymptotic variance · Second-order asymptotic loss

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