

## Intrinsically weighted means and non-ergodic marked point processes

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Abstract Mean marks form a versatile toolbox in the analysis of marked point processes (MPPs). For ergodic processes, their definition is straightforward and practical application is well established. In the stationary non-ergodic case, though, different definitions of mark averages are possible and might be practically relevant. In this paper, the classical definition of mean marks is compared to a set of new characteristics for non-ergodic MPPs, which stand out due to the weighting of ergodicity classes. Another weighting can be introduced on the single-point level via weights given by the marks themselves. These intrinsically given weights and the weighting of ergodicity classes are closely related to each other meaning that for suitable choices of weights, a mean mark characteristic can be expressed in either way. Estimators for the different definitions of mean marks are discussed and their consistency and asymptotic normality are shown under certain conditions.

Keywords Ergodic decomposition  $\cdot$  Hierarchical modeling  $\cdot$  Mark-location interaction  $\cdot$  Moment measure  $\cdot$  Non-ergodicity  $\cdot$  Weighted mark mean