

# On consistency and optimality of Bayesian variable selection based on $g$ -prior in normal linear regression models

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Received: 7 August 2013 / Revised: 22 May 2014 / Published online: 24 September 2014  
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**Abstract** Consider Bayesian variable selection in normal linear regression models based on Zellner’s  $g$ -prior. We study theoretical properties of this method when the sample size  $n$  grows and consider the cases when the number of regressors,  $p$  is fixed and when it grows with  $n$ . We first consider the situation where the true model is not in the model space and prove under mild conditions that the method is consistent and “loss efficient” in appropriate sense. We then consider the case when the true model is in the model space and prove that the posterior probability of the true model goes to one as  $n$  goes to infinity. “Loss efficiency” is also proved in this situation. We give explicit conditions on the rate of growth of  $g$ , possibly depending on that of  $p$  as  $n$  grows, for our results to hold. This helps in making recommendations for the choice of  $g$ .

**Keywords** Model selection consistency · Loss efficiency · Variable selection ·  $g$ -prior