

## On the equivariance criterion in statistical prediction

Haojin Zhou · Tapan K. Nayak

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**Abstract** This paper presents a general development of the basic logic of equivariance for a parametric point prediction problem. We propose a framework that allows the set of possible predictions as well as the losses to depend on the data and then explore the nature and properties of relevant transformation groups for applying the functional and formal equivariance principles. We define loss invariance and predictive equivariance appropriately and discuss their ramifications. We describe a structure of equivariant predictors in terms of maximal invariants and present a method for deriving minimum risk equivariant predictors. We explore the connections between equivariance and risk unbiasedness and show that uniquely best risk unbiased predictors are almost equivariant. We apply our theoretical results to some illustrative examples.

**Keywords** Loss invariance · Maximal invariant · Minimum risk · Risk unbiased · Transformation group