

## Estimation of a non-negative location parameter with unknown scale

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**Abstract** For a vast array of general spherically symmetric location-scale models with a residual vector, we consider estimating the (univariate) location parameter when it is lower bounded. We provide conditions for estimators to dominate the benchmark minimax MRE estimator, and thus be minimax under scale invariant loss. These minimax estimators include the generalized Bayes estimator with respect to the truncation of the common non-informative prior onto the restricted parameter space for normal models under general convex symmetric loss, as well as non-normal models under scale invariant  $L^p$  loss with  $p > 0$ . We cover many other situations when the loss is asymmetric, and where other generalized Bayes estimators, obtained with different powers of the scale parameter in the prior measure, are proven to be minimax. We rely on various novel representations, sharp sign change analyses, as well as capitalize on Kubokawa's integral expression for risk difference technique. Several properties such as robustness of the generalized Bayes estimators under various loss functions are obtained.

**Keywords** Dominance · Generalized Bayes · Lower bounded mean ·  $L^p$  loss · Minimax · Robustness