Permissible boundary prior function as a virtually proper prior density

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Abstract Regularity conditions for an improper prior function to be regarded as a virtually proper prior density are proposed, and their implications are discussed. The two regularity conditions require that a prior function is defined as a limit of a sequence of proper prior densities and also that the induced posterior density is derived as a smooth limit of the sequence of corresponding posterior densities. This approach is compared with the assumption of a degenerated prior density at an unknown point, which is familiar in the empirical Bayes method. The comparison study extends also to the assumption of an improper prior function discussed separately from any proper prior density. Properties and examples are presented to claim potential usefulness of the proposed notion.

Keywords Degenerated prior · Logarithmic divergence · Marginal density · Non-informative prior · Weakly informative prior