Qualitative inequalities for squared partial correlations of a Gaussian random vector

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Abstract We describe various sets of conditional independence relationships, sufficient for qualitatively comparing non-vanishing squared partial correlations of a Gaussian random vector. These sufficient conditions are satisfied by several graphical Markov models. Rules for comparing degree of association among the vertices of such Gaussian graphical models are also developed. We apply these rules to compare conditional dependencies on Gaussian trees. In particular for trees, we show that such dependence can be completely characterised by the length of the paths joining the dependent vertices to each other and to the vertices conditioned on. We also apply our results to postulate rules for model selection for polytree models. Our rules apply to mutual information of Gaussian random vectors as well.

Keywords Inequalities \cdot Graphical Markov models \cdot Mutual information \cdot Squared partial correlation \cdot Tree models