

# Asymptotic distribution of the nonparametric distribution estimator based on a martingale approach in doubly censored data

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**Abstract** For analysis of time-to-event data with incomplete information beyond right-censoring, many generalizations of the inference of the distribution and regression model have been proposed. However, the development of martingale approaches in this area has not progressed greatly, while for right-censored data such an approach has spread widely to study the asymptotic properties of estimators and to derive regression diagnosis methods. In this paper, focusing on doubly censored data, we discuss a martingale approach for inference of the nonparametric maximum likelihood estimator (NPMLE). We formulate a martingale structure of the NPMLE using a score function of the semiparametric profile likelihood. Finally, an expression of the asymptotic distribution of the NPMLE is derived more conveniently without depending on an infinite matrix expression as in previous research. A further useful point is that a variance-covariance formula of the NPMLE computable in a larger sample is obtained as an empirical version of the limit form presented here.

**Keywords** Counting process · Forward-backward intensities · Nonparametric maximum likelihood estimator · Semiparametric profile likelihood · Weak convergence