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# An optimal modification of the LIML estimation for many instruments and persistent heteroscedasticity

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**Abstract** We consider the estimation of coefficients of a structural equation with many instrumental variables in a simultaneous equation system. It is mathematically equivalent to the *estimating equations estimation* or a *reduced rank regression* in the statistical multivariate linear models when the number of restrictions or the dimension of estimating equations increases with the sample size. As a semi-parametric method, we propose a class of modifications of the limited information maximum likelihood (LIML) estimator to improve its asymptotic properties as well as the small sample properties for many instruments and persistent heteroscedasticity. We show that an asymptotically optimal modification of the LIML estimator, which is called AOM-LIML, improves the LIML estimator and other estimation methods. We give a set of sufficient conditions for an asymptotic optimality when the number of instruments or the dimension of the estimating equations is large with persistent heteroscedasticity including a case of *many weak instruments*.

**Keywords** Estimation of structural equation · Estimating equation estimation · Reduced rank regression · Many instruments · Persistent heteroscedasticity · AOM-LIML · Asymptotic optimality