

# Strong consistency of nonparametric Bayes density estimation on compact metric spaces with applications to specific manifolds

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**Abstract** This article considers a broad class of kernel mixture density models on compact metric spaces and manifolds. Following a Bayesian approach with a nonparametric prior on the location mixing distribution, sufficient conditions are obtained on the kernel, prior and the underlying space for strong posterior consistency at any continuous density. The prior is also allowed to depend on the sample size  $n$  and sufficient conditions are obtained for weak and strong consistency. These conditions are verified on compact Euclidean spaces using multivariate Gaussian kernels, on the hypersphere using a von Mises-Fisher kernel and on the planar shape space using complex Watson kernels.

**Keywords** Nonparametric Bayes · Density estimation · Posterior consistency · Sample-dependent prior · Riemannian manifold · Hypersphere · Shape space