

Statistical modeling for discrete patterns in a sequence of exchangeable trials

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Abstract This paper proposes a new method for constructing a sequence of infinitely exchangeable uniform random variables on the unit interval. For constructing the sequence, we utilize a Pólya urn partially. The resulting exchangeable sequence depends on the initial numbers of balls of the Pólya urn. We also derive the de Finetti measure for the exchangeable sequence. For an arbitrarily given one-dimensional distribution function, we generate sequences of exchangeable random variables with the one-dimensional marginal distribution by transforming the exchangeable uniform sequences with the inverse function of the distribution function. Among them we mainly investigate sequences of exchangeable discrete random variables. They differ from the well-known exchangeable sequence generated only by the Pólya urn scheme. Some examples are also given as applications of the results to exact distributions of some statistics based on sequences of exchangeable trials. Further, from the above exchangeable uniform sequence we construct partial or Markov exchangeable sequences. We also provide numerical examples of statistical inference based on the exchangeable and Markov exchangeable sequences.

Keywords Discrete pattern · de Finetti measure · Exchangeability · Waiting time · Generalized probability generating function · Pólya's urn · Beta distribution · Conditional expectation · Discrete distribution theory · Partial exchangeability