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Isoseparation and robustness in parametric Bayesian inference

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Abstract This paper introduces a new family of local density separations for assessing robustness of finite-dimensional Bayesian posterior inferences with respect to their priors. Unlike for their global equivalents, under these novel separations posterior robustness is recovered even when the functioning posterior converges to a defective distribution, irrespectively of whether the prior densities are grossly misspecified and of the form and the validity of the assumed data sampling distribution. For exponential family models, the local density separations are shown to form the basis of a weak topology closely linked to the Euclidean metric on the natural parameters. In general, the local separations are shown to measure relative roughness of the prior distribution with respect to its corresponding posterior and provide explicit bounds for the total variation distance between an approximating posterior density to a genuine posterior. We illustrate the application of these bounds for assessing robustness of the posterior inferences for a dynamic time series model of blood glucose concentration in diabetes mellitus patients with respect to alternative prior specifications.

Keywords Density ratio class · Hierarchical Bayesian inference · Local robustness · Total variation · Power steady model · Diabetes mellitus