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Uniform in bandwidth exact rates for a class of kernel estimators

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Abstract Given an i.i.d sample (Y_i, Z_i), taking values in $\mathbb{R}^{d'} \times \mathbb{R}^d$, we consider a collection Nadarya–Watson kernel estimators of the conditional expectations $\mathbb{E}(< c_g(z), g(Y) > +d_g(z) \mid Z = z)$, where z belongs to a compact set $H \subset \mathbb{R}^d$, g a Borel function on $\mathbb{R}^{d'}$ and $c_g(\cdot), d_g(\cdot)$ are continuous functions on $\mathbb{R}^d$. Given two bandwidth sequences $h_n < \mathfrak{h}_n$ fulfilling mild conditions, we obtain an exact and explicit almost sure limit bounds for the deviations of these estimators around their expectations, uniformly in $g \in \mathcal{G},\ z \in H$ and $h_n \le h \le \mathfrak{h}_n$ under mild conditions on the density f_Z, the class $\mathcal{G}$, the kernel K and the functions $c_g(\cdot), d_g(\cdot)$. We apply this result to prove that smoothed empirical likelihood can be used to build confidence intervals for conditional probabilities $\mathbb{P}(Y \in C \mid Z = z)$, that hold uniformly in $z \in H,\ C \in \mathcal{C},\ h \in [h_n, \mathfrak{h}_n]$. Here $\mathcal{C}$ is a Vapnik–Chervonenkis class of sets.

Keywords Local empirical processes · Empirical likelihood · Kernel smoothing · Uniform in bandwidth consistency