

## On Bayes inference for a bathtub failure rate via $\mathbf{S}$ -paths

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**Abstract** A class of semi-parametric hazard/failure rates with a bathtub shape is of interest. It does not only provide a great deal of flexibility over existing parametric methods in the modeling aspect but also results in a closed-form and tractable Bayes estimator for the bathtub-shaped failure rate. Such an estimator is derived to be a finite sum over two  $\mathbf{S}$ -paths due to an explicit posterior analysis in terms of two (conditionally independent)  $\mathbf{S}$ -paths. These, newly discovered, explicit results can be proved to be Rao–Blackwell improvements of counterpart results in terms of partitions that are readily available by a specialization of James’ work (Ann Stat 33:1771–1799, 2005). Both iterative and non-iterative computational procedures are introduced for evaluating the hazard estimates. Two applications of the proposed methodology are discussed, of which one is about a Bayesian test for bathtub-shaped failure rates and the other is related to modeling with covariates.

**Keywords** Random partition · Completely random measure · Rao–Blackwellization · Sequential importance sampling · Gibbs sampler · Accelerated path sampler