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Limiting size index distributions for ball-bin models with Zipf-type frequencies

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Abstract We consider a random ball-bin model where balls are thrown randomly and sequentially into a set of bins. The frequency of choices of bins follows the Zipf-type (power-law) distribution; that is, the probability with which a ball enters the ith most popular bin is asymptotically proportional to $1/i^{\alpha}$, $\alpha > 0$. In this model, we derive the limiting size index distributions to which the empirical distributions of size indices converge almost surely, where the size index of degree k at time t represents the number of bins containing exactly k balls at t. While earlier studies have only treated the case where the power α of the Zipf-type distribution is greater than unity, we here consider the case of $\alpha \leq 1$ as well as $\alpha > 1$. We first investigate the limiting size index distributions for the independent throw models and then extend the derived results to a case where bins are chosen dependently. Simulation experiments demonstrate not only that our analysis is valid but also that the derived limiting distributions well approximate the empirical size index distributions in a relatively short period.

Keywords Limiting distributions · Random ball-bin occupancy models · Size indices · Zipf-type distribution