

Asymptotic properties of sample quantiles of discrete distributions

Yanyuan Ma · Marc G. Genton · Emanuel Parzen

Received: 23 May 2008 / Revised: 14 October 2008 / Published online: 22 January 2009
© The Institute of Statistical Mathematics, Tokyo 2009

Abstract The asymptotic distribution of sample quantiles in the classical definition is well-known to be normal for absolutely continuous distributions. However, this is no longer true for discrete distributions or samples with ties. We show that the definition of sample quantiles based on mid-distribution functions resolves this issue and provides a unified framework for asymptotic properties of sample quantiles from absolutely continuous and from discrete distributions. We demonstrate that the same asymptotic normal distribution result as for the classical sample quantiles holds at differentiable points, whereas a more general form arises for distributions whose cumulative distribution function has only one-sided differentiability. For discrete distributions with finite support, the same type of asymptotics holds and the sample quantiles based on mid-distribution functions either follow a normal or a two-piece normal distribution. We also calculate the exact distribution of these sample quantiles for the binomial and Poisson distributions. We illustrate the asymptotic results with simulations.

Keywords Asymptotic · Continuous · Discrete · Grouped data · Mid-distribution · Quantile · Ties