

Gambler's ruin and winning a series by m games

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Abstract Two teams play a series of games until one team accumulates m more wins than the other. These series are fairly common in some sports provided that the competition has already extended beyond some number of games. We generalize these schemes to allow ties in the single games. Different approaches offer different advantages in calculating the winning probabilities and the distribution of the duration N , including difference equations, conditioning, explicit and implicit path counting, generating functions and a martingale-based derivation of the probability and moment generating functions of N . The main result of the paper is the determination of the exact distribution of N for a series of fair games without ties as a sum of independent geometrically distributed random variables and its approximation.

Keywords Gambler's ruin · Distribution of the duration · Martingales · Probability and moment generating functions · Limit theorem · Chebyshev polynomial of the first kind