

## Optimal tuning parameter estimation in maximum penalized likelihood method

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**Abstract** In maximum penalized or regularized methods, it is important to select a tuning parameter appropriately. This paper proposes a direct plug-in method for tuning parameter selection. The tuning parameters selected using a generalized information criterion (Konishi and Kitagawa, *Biometrika*, 83, 875–890, 1996) and cross-validation (Stone, *Journal of the Royal Statistical Society, Series B*, 58, 267–288, 1974) are shown to be asymptotically equivalent to those selected using the proposed method, from the perspective of estimation of an optimal tuning parameter. Because of its directness, the proposed method is superior to the two selection methods mentioned above in terms of computational cost. Some numerical examples which contain the penalized spline generalized linear model regressions are provided.

**Keywords** Cross-validation · Direct plug-in method · Generalized information criterion · Kullback–Leibler information · Maximum penalized likelihood method · Penalized spline · Ridge regression · Tuning parameter estimation