Improved prediction for a multivariate normal distribution with unknown mean and variance

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Abstract The prediction problem for a multivariate normal distribution is considered where both mean and variance are unknown. When the Kullback–Leibler loss is used, the Bayesian predictive density based on the right invariant prior, which turns out to be a density of a multivariate *t*-distribution, is the best invariant and minimax predictive density. In this paper, we introduce an improper shrinkage prior and show that the Bayesian predictive density against the shrinkage prior improves upon the best invariant predictive density when the dimension is greater than or equal to three.

Keywords Bayesian prediction \cdot Kullback–Leibler divergence \cdot Multivariate normal distribution \cdot Multivariate *t*-distribution \cdot Right invariant prior \cdot Shrinkage prior \cdot Star ordering