

On a simple strategy weakly forcing the strong law of large numbers in the bounded forecasting game

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Received: 2 August 2006 / Revised: 6 December 2006 / Published online: 9 March 2007
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Abstract In the framework of the game-theoretic probability of Shafer and Vovk it is of basic importance to construct an explicit strategy weakly forcing the strong law of large numbers in the bounded forecasting game. We present a simple finite-memory strategy based on the past average of Reality's moves, which weakly forces the strong law of large numbers with the convergence rate of $O(\sqrt{\log n/n})$. Our proof is very simple compared to a corresponding measure-theoretic result of Azuma (*The Tôhoku Mathematical Journal*, 19, 357–367, 1967) on bounded martingale differences and this illustrates effectiveness of game-theoretic approach. We also discuss one-sided protocols and extension of results to linear protocols in general dimension.

Keywords Azuma-Hoeffding-Bennett inequality · Capital process · Game-theoretic probability · Large deviation