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Higher order estimation at Lebesgue points

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Abstract The symmetric derivative of a probability measure at a Lebesgue point can often be specified by an exact relation involving a regularity index. Knowledge of this index is of practical interest, for example to specify the local behavior of the measure under study and to evaluate bandwidths or number of neighbors to take into account in smoothing techniques. This index also determines local rates of convergence of estimators of particular points of curves and surfaces, like minima and maxima. In this paper, we consider the estimation of the d-dimensional regularity index. We introduce an estimator and derive the basic asymptotic results. Our estimator is inspired by an estimator proposed in Drees and Kaufmann (1998, *Stochastic Processes and their Applications*, *75*, 149–172) in the context of extreme value statistics. Then, we show how (estimates of) the regularity index can be used to solve practical problems in nearest neighbor density estimation, such as removing bias or selecting the number of neighbors. Results of simulations are presented.

Keywords Lebesgue point · Mode estimation · Nearest neighbor density estimation · Probability density · Rate of convergence · Regularity index