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Minimal invariant Markov basis for sampling contingency tables with fixed marginals

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Abstract In this paper we define an invariant Markov basis for a connected Markov chain over the set of contingency tables with fixed marginals and derive some characterizations of minimality of the invariant basis. We also give a necessary and sufficient condition for uniqueness of minimal invariant Markov bases. By considering the invariance, Markov bases can be presented very concisely. As an example, we present minimal invariant Markov bases for all $2 \times 2 \times 2 \times 2$ hierarchical models. The invariance here refers to permutation of indices of each axis of the contingency tables. If the categories of each axis do not have any order relations among them, it is natural to consider the action of the symmetric group on each axis of the contingency table. A general algebraic algorithm for obtaining a Markov basis was given by Diaconis and Sturmfels (*The Annals of Statistics*, **26**, 363–397, 1998). Their algorithm is based on computing Gröbner basis of a well-specified polynomial ideal. However, the reduced Gröbner basis depends on the particular term order and is not symmetric. Therefore, it is of interest to consider the properties of invariant Markov basis.

Keywords Exact tests · Hierarchical models · Markov chain Monte Carlo · Orbit · Symmetric group · Transformation group