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A first–passage time random walk distribution with five transition probabilities: a generalization of the shifted inverse trinomial

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Abstract In this paper a univariate discrete distribution, denoted by GIT, is proposed as a generalization of the shifted inverse trinomial distribution, and is formulated as a first-passage time distribution of a modified random walk on the half-plane with five transition probabilities. In contrast, the inverse trinomial arises as a random walk on the real line with three transition probabilities. The probability mass function (pmf) is expressible in terms of the Gauss hypergeometric function and this offers computational advantage due to its recurrence formula. The descending factorial moment is also obtained. The GIT contains twenty-two possible distributions in total. Special cases include the binomial, negative binomial, shifted negative binomial, shifted inverse binomial or, equivalently, lost-games, and shifted inverse trinomial distributions. A subclass $\mathrm{GIT}_{3,1}$ is a particular member of Kemp's class of convolution of pseudo-binomial variables and its properties such as reproductivity, formulation, pmf, moments, index of dispersion, and approximations are studied in detail. Compound or generalized (stopped sum) distributions provide inflated models. The inflated $\mathrm{GIT}_{3,1}$ extends Minkova's inflated-parameter binomial and negative binomial. A bivariate model which has the GIT as a marginal distribution is also proposed.

Keywords Binomial · Inflated model · Negative binomial · Random walk · Shifted inverse binomial · Shifted negative binomial