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Estimation of a multivariate normal covariance matrix with staircase pattern data

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Abstract In this paper, we study the problem of estimating a multivariate normal covariance matrix with staircase pattern data. Two kinds of parameterizations in terms of the covariance matrix are used. One is Cholesky decomposition and another is Bartlett decomposition. Based on Cholesky decomposition of the covariance matrix, the closed form of the maximum likelihood estimator (MLE) of the covariance matrix is given. Using Bayesian method, we prove that the best equivariant estimator of the covariance matrix with respect to the special group related to Cholesky decomposition uniquely exists under the Stein loss. Consequently, the MLE of the covariance matrix is inadmissible under the Stein loss. Our method can also be applied to other invariant loss functions like the entropy loss and the symmetric loss. In addition, based on Bartlett decomposition of the covariance matrix, the Jeffreys prior and the reference prior of the covariance matrix with staircase pattern data are also obtained. Our reference prior is different from Berger and Yang's reference prior. Interestingly, the Jeffreys prior with staircase pattern data is the same as that with complete data. The posterior properties are also investigated. Some simulation results are given for illustration.

Keywords Maximum likelihood estimator · Best equivariant estimator · Covariance matrix · Staircase pattern data · Invariant Haar measure · Cholesky decomposition · Bartlett decomposition · Inadmissibility · Jeffreys prior · Reference prior