

A note on curvature of α -connections of a statistical manifold

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Abstract The family of α -connections $\nabla^{(\alpha)}$ on a statistical manifold \mathcal{M} equipped with a pair of conjugate connections $\nabla \equiv \nabla^{(1)}$ and $\nabla^* \equiv \nabla^{(-1)}$ is given as $\nabla^{(\alpha)} = \frac{1+\alpha}{2}\nabla + \frac{1-\alpha}{2}\nabla^*$. Here, we develop an expression of curvature $R^{(\alpha)}$ for $\nabla^{(\alpha)}$ in relation to those for ∇, ∇^* . Immediately evident from it is that $\nabla^{(\alpha)}$ is equiaffine for any $\alpha \in \mathbb{R}$ when ∇, ∇^* are dually flat, as previously observed in Takeuchi and Amari (*IEEE Transactions on Information Theory* 51:1011–1023, 2005). Other related formulae are also developed.

Keywords Equiaffine connections · Parallel volume form · Ricci tensor · Cubic/skewness form