

Exponential statistical manifold

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Abstract We consider the non-parametric statistical model $\mathcal{E}(p)$ of all positive densities q that are connected to a given positive density p by an open exponential arc, i.e. a one-parameter exponential model $p(t)$, $t \in I$, where I is an open interval. On this model there exists a manifold structure modeled on Orlicz spaces, originally introduced in 1995 by Pistone and Sempi. Analytic properties of such a manifold are discussed. Especially, we discuss the regularity of mixture models under this geometry, as such models are related with the notion of e- and m-connections as discussed by Amari and Nagaoka.

Keywords Information geometry · Statistical manifold · Orlicz space · Moment generating functional · Cumulant generating functional · Kullback–Leibler divergence