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## Generalized moment theory and Bayesian robustness analysis for hierarchical mixture models

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Abstract In applications of Bayesian analysis one problem that arises is the evaluation of the sensitivity, or robustness, of the adopted inferential procedure with respect to the components of the formulated statistical model. In particular, it is of interest to study robustness with respect to the prior, when this latter cannot be uniquely elicitated, but a whole class  $\Gamma$  of probability measures, agreeing with the available information, can be identified. In this situation, the analysis of robustness consists of finding the extrema of posterior functionals under  $\Gamma$ . In this paper, we provide a theoretical framework for the treatment of a global robustness problem in the context of hierarchical mixture modeling, where the mixing distribution is a random probability whose law belongs to a generalized moment class  $\Gamma$ . Under suitable conditions on the functions describing the problem, the solution of this latter coincides with the solution of a linear semi-infinite programming problem.

**Keywords** Bayesian robustness analysis · Hierarchical mixture models · Nonparametric prior · Moment theory · Linear semi-infinite programming