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# Semiparametric maximum likelihood for missing covariates in parametric regression

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**Abstract** We consider parameter estimation in parametric regression models with covariates missing at random. This problem admits a semiparametric maximum likelihood approach which requires no parametric specification of the selection mechanism or the covariate distribution. The semiparametric maximum likelihood estimator (MLE) has been found to be consistent. We show here, for some specific models, that the semiparametric MLE converges weakly to a zero-mean Gaussian process in a suitable space. The regression parameter estimate, in particular, achieves the semiparametric information bound, which can be consistently estimated by perturbing the profile log-likelihood. Furthermore, the profile likelihood ratio statistic is asymptotically chi-squared. The techniques used here extend to other models.

**Keywords** Asymptotic normality · Efficiency · Infinite-dimensional M-estimation · Missing at random · Missing covariates · Parametric regression · Profile likelihood · Semiparametric likelihood