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# Waiting time distributions of simple and compound patterns in a sequence of $r$ -th order Markov dependent multi-state trials

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**Abstract** Waiting time distributions of simple and compound runs and patterns have been studied and applied in various areas of statistics and applied probability. Most current results are derived under the assumption that the sequence is either independent and identically distributed or first order Markov dependent. In this manuscript, we provide a comprehensive theoretical framework for obtaining waiting time distributions under the most general structure where the sequence consists of  $r$ -th order ( $r \geq 1$ ) Markov dependent multi-state trials. We also show that the waiting time follows a generalized geometric distribution in terms of the essential transition probability matrix of the corresponding imbedded Markov chain. Further, we derive a large deviation approximation for the tail probability of the waiting time distribution based on the largest eigenvalue of the essential transition probability matrix. Numerical examples are given to illustrate the simplicity and efficiency of the theoretical results.

**Keywords** Runs and patterns · Markov chain imbedding · Transition probability matrix · Large deviation approximation