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Testing for tail independence in extreme value models

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Abstract Let (X, Y) be a random vector which follows in its upper tail a bivariate extreme value distribution with reverse exponential margins. We show that the conditional distribution function (df) of $X + Y$, given that $X + Y > c$, converges to the df $F(t) = t^2$, $t \in [0, 1]$, as $c \uparrow 0$ if and only if X, Y are tail independent. Otherwise, the limit is $F(t) = t$. This is utilized to test for the tail independence of X, Y via various tests, including the one suggested by the Neyman–Pearson lemma. Simulations show that the Neyman–Pearson test performs best if the threshold c is close to 0, whereas otherwise it is the Kolmogorov–Smirnov test that performs best. The mathematical conditions are studied under which the Neyman–Pearson approach actually controls the type I error. Our considerations are extended to extreme value distributions in arbitrary dimensions as well as to distributions which are in a differentiable spectral neighborhood of an extreme value distribution.

Keywords Bivariate extremes · Pickands dependence function · Tail independence · Tail dependence parameter · Neyman–Pearson test · Kolmogorov–Smirnov test · Fisher's κ · Chi-square goodness-of-fit test · Differentiable spectral neighborhood · Generalized Pareto distribution