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Exact and limiting distributions in diagonal Pólya processes

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Abstract We investigate the Pólya process, which underlies an urn of white and blue balls growing in real time. A partial differential equation governs the evolution of the process. For urns with (forward or backward) diagonal ball addition matrix the partial differential equation is amenable to asymptotic solution. In the case of forward diagonal we find a solution via the method of characteristics; the numbers of white and blue balls, when scaled appropriately, converge in distribution to independent Gamma random variables. The method of characteristics becomes a bit too involved for the backward diagonal process, except in degenerate cases, where we have Poisson behavior. In nondegenerate cases, limits characterized implicitly by their recursive sequence of moments are found, via matrix formulation involving a Leonard pair.

Keywords Urns · Random structure · Stochastic process · Partial differential equation · Leonard pairs