

NECESSARY CONDITIONS FOR DOMINATING THE JAMES-STEIN ESTIMATOR

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Abstract. This paper develops necessary conditions for an estimator to dominate the James-Stein estimator and hence the James-Stein positive-part estimator. The ultimate goal is to find classes of such dominating estimators which are admissible. While there are a number of results giving classes of estimators dominating the James-Stein estimator, the only admissible estimator known to dominate the James-Stein estimator is the generalized Bayes estimator relative to the fundamental harmonic function in three and higher dimension. The prior was suggested by Stein and the domination result is due to Kubokawa. Shao and Strawderman gave a class of estimators dominating the James-Stein positive-part estimator but were unable to demonstrate admissibility of any in their class. Maruyama, following a suggestion of Stein, has studied generalized Bayes estimators which are members of a point mass at zero and a prior similar to the harmonic prior. He finds a subclass which is minimax and admissible but is unable to show that any in his class with positive point mass at zero dominate the James-Stein estimator. The results in this paper show that a subclass of Maruyama's procedures including the class that Stein conjectured might contain members dominating the James-Stein estimator cannot dominate the James-Stein estimator. We also show that under reasonable conditions, the "constant" in shrinkage factor must approach $p - 2$ for domination to hold.

Key words and phrases: The James-Stein estimator, unbiased estimator of risk, admissibility, generalized Bayes.