UNIVERSALLY CONSISTENT CONDITIONAL U-STATISTICS FOR ABSOLUTELY REGULAR PROCESSES AND ITS APPLICATIONS FOR HIDDEN MARKOV MODELS*

Michel Harel^1 and $\mathsf{Madan}\ \mathsf{L}.\ \mathsf{Purl}^2$

¹ UMRC 55830 C.N.R.S. and IUFM du Limousin, 209 Bd de Vanteaux, F87036 Limoges Cedex, France, e-mail: harel@unilim.fr ² Department of Mathematics, Indiana University, Bloomington, IN 47405, U.S.A.

(Received November 18, 2002; revised November 11, 2003)

Abstract. A general class of conditional U-statistics was introduced by W. Stute as a generalization of the Nadaraya-Watson estimates of a regression function. It was shown that such statistics are universally consistent. Also, universal consistencies of the window and k_n -nearest neighbor estimators (as two special cases of the conditional U-statistics) were proved. In this paper, we extend these results from the independent case to dependent case. The result is applied to verify the Bayes risk consistency of the corresponding discrimination rules.

Key words and phrases: Universally consistent conditional *U*-statistics, absolute regularity, Bayes risk, Hidden Markov Models.

^{*}Research supported by the Office of Naval Research Contract N00014-91-J-1020.