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TYPES OF LIKELIHOOD MAXIMA IN MIXTURE MODELS AND THEIR IMPLICATION ON THE PERFORMANCE OF TESTS*

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Abstract. In two-component mixtures of exponential distributions, different strategies for starting the likelihood maximization algorithm converge to different types of maxima. The power of an LR test of homogeneity against such a mixture strongly depends on the considered strategy, and global maximization need not result in the largest power. An explanation is given on basis of a systematic investigation of the likelihood function in a large number of simulations, using a variety of diagnostic tools. Thereby, we also gain a deeper insight into the properties of the samples that generate particular types of solutions of the likelihood equation. In particular, "spurious solutions" often occur; these are mainly responsible for the fact that global maximization may not result in a statistically meaningful estimator. Removing the smallest elements of a sample may drastically increase the power of previously inferior strategies.

Key words and phrases: Mixture models, likelihood function, likelihood ratio tests, multiple maxima, likelihood equation, spurious solutions, EM algorithm, starting values.

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