

JOINT DISTRIBUTIONS ASSOCIATED WITH PATTERNS, SUCCESSES AND FAILURES IN A SEQUENCE OF MULTI-STATE TRIALS*

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Abstract. Let $\{Z_t, t \geq 1\}$ be a sequence of trials taking values in a given set $\mathcal{A} = \{0, 1, 2, \dots, m\}$, where we regard the value 0 as failure and the remaining m values as successes. Let \mathcal{E} be a (single or compound) pattern. In this paper, we provide a unified approach for the study of two joint distributions, i.e., the joint distribution of the number X_n of occurrences of \mathcal{E} , the numbers of successes and failures in n trials and the joint distribution of the waiting time T_r until the r -th occurrence of \mathcal{E} , the numbers of successes and failures appeared at that time. We also investigate some distributions as by-products of the two joint distributions. Our methodology is based on two types of the random variables X_n (a Markov chain imbeddable variable of binomial type and a Markov chain imbeddable variable of returnable type). The present work develops several variations of the Markov chain imbedding method and enables us to deal with the variety of applications in different fields. Finally, we discuss several practical examples of our results.

Key words and phrases: Run, pattern, waiting time, enumeration schemes, Markov chain, double generating function, probability generating function, Markov chain imbedding method, transition probability matrices.

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