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A NEW CLASS OF METRIC DIVERGENCES ON PROBABILITY SPACES AND ITS APPLICABILITY IN STATISTICS

FERDINAND ÖSTERREICHER[1] AND IGOR VAJDA[2]*

[1]*Institute of Mathematics, University of Salzburg, 5020 Salzburg, Austria*
[2]*Institute of Information Theory and Automation, Academy of Sciences, 18208 Prague, Czech Republic*

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Abstract. The class I_{f_β}, $\beta \in (0, \infty]$, of f-divergences investigated in this paper is defined in terms of a class of entropies introduced by Arimoto (1971, *Information and Control*, **19**, 181–194). It contains the squared Hellinger distance (for $\beta = 1/2$), the sum $I(Q_1\|(Q_1+Q_2)/2)+I(Q_2\|(Q_1+Q_2)/2)$ of Kullback-Leibler divergences (for $\beta = 1$) and half of the variation distance (for $\beta = \infty$) and continuously extends the class of squared perimeter-type distances introduced by Österreicher (1996, *Kybernetika*, **32**, 389–393) (for $\beta \in (1, \infty]$). It is shown that $(I_{f_\beta}(Q_1, Q_2))^{\min(\beta,1/2)}$ are distances of probability distributions Q_1, Q_2 for $\beta \in (0, \infty)$. The applicability of I_{f_β}-divergences in statistics is also considered. In particular, it is shown that the I_{f_β}-projections of appropriate empirical distributions to regular families define distribution estimates which are in the case of an i.i.d. sample of size n consistent. The order of consistency is investigated as well.

Key words and phrases: Dissimilarities, metric divergences, minimum distance estimators.

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