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ON DISCRETE α-UNIMODALITY

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Abstract. A continuous composition semigroup of probability generating functions $F := (F_t, t \geq 0)$ and the corresponding multiplication $\odot_F$ of van Harn *et al.* (1982, *Z. Wahrsch. Verw. Gebiete*, **61**, 97–118) are used to introduce the concept of $[F;\alpha]$-unimodality which generalizes the discrete α-unimodality due to Abouammoh (1987, *Statist. Neerlandica*, **41**, 239–244) and Alamatsaz (1993, *Statist. Neerlandica*, **47**, 245–252). We offer various characterizations and other properties of $[F;\alpha]$-unimodality. Notably, several convolution results are presented. Moreover, we explore the relationship between $[F;\alpha]$-unimodality and the concepts of discrete self-decomposability and stability. Finally, lower bounds for variances of $[F;\alpha]$-monotone and $[F;\alpha]$-unimodal random variables are derived and some examples are also mentioned.

Key words and phrases: Lattice distribution, semigroup, monotonicity, generating function, mixture, convolution, variance bounds.

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