

## ASYMPTOTIC BOUNDS FOR ESTIMATORS WITHOUT LIMIT DISTRIBUTION

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**Abstract.** Let  $\mathfrak{P}$  be a general family of probability measures,  $\kappa : \mathfrak{P} \rightarrow \mathbb{R}$  a functional, and  $N_{(0, \sigma^2(P))}$  the optimal limit distribution for regular estimator sequences of  $\kappa$ . On intervals symmetric about 0, the concentration of this optimal limit distribution can be surpassed by the asymptotic concentration of an arbitrary estimator sequence only for  $P$  in a “small” subset of  $\mathfrak{P}$ . For asymptotically median unbiased estimator sequences the same is true for arbitrary intervals containing 0. The emphasis of the paper is on “pointwise” conditions for  $P \in \mathfrak{P}$ , as opposed to conditions on shrinking neighbourhoods, and on “general” rather than parametric families.

*Key words and phrases:* Estimation, asymptotic theory, local uniformity, nonparametric theory, minimax bounds.