UNBIASED BAYES ESTIMATES AND IMPROPER PRIORS

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Abstract. Given two random variables (X, Y) the condition of unbiasedness states that: $E(X \mid Y = y) = y$ and $E(Y \mid X = x) = x$ both almost surely (a.s.). If the prior on Y is proper and has finite expectation or non-negative support, unbiasedness implies $X = Y$ a.s. This paper examines the implications of unbiasedness when the prior on Y is improper. Since the improper case can be meaningfully analysed in a finitely additive framework, we revisit the whole issue of unbiasedness from this perspective. First we argue that a notion weaker than equality a.s., named coincidence, is more appropriate in a finitely additive setting. Next we discuss the meaning of unbiasedness from a Bayesian and fiducial perspective. We then show that unbiasedness and finite expectation of Y imply coincidence between X and Y, while a weaker conclusion follows if the improper prior on Y is only assumed to have positive support. We illustrate our approach throughout the paper by revisiting some examples discussed in the recent literature.

Key words and phrases: Coincidence, dF-coherence, equality almost surely, finite additivity, improper prior, unbiasedness.