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BOOTSTRAP CHOICE OF TUNING PARAMETERS

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Abstract. Consider the problem of estimating $\theta = \theta(P)$ based on data x_n from an unknown distribution P. Given a family of estimators $T_{n,\beta}$ of $\theta(P)$, the goal is to choose β among $\beta \in I$ so that the resulting estimator is as good as possible. Typically, β can be regarded as a tuning or smoothing parameter, and proper choice of β is essential for good performance of $T_{n,\beta}$. In this paper, we discuss the theory of β being chosen by the bootstrap. Specifically, the bootstrap estimate of β, $\hat{\beta}_n$, is chosen to minimize an empirical bootstrap estimate of risk. A general theory is presented to establish the consistency and weak convergence properties of these estimators. Confidence intervals for $\theta(P)$ based on $T_{n,\hat{\beta}_n}$ are also asymptotically valid. Several applications of the theory are presented, including optimal choice of trimming proportion, bandwidth selection in density estimation and optimal combinations of estimates.

Key words and phrases: Bandwidth selection, bootstrap, confidence limits, density estimation, risk function.