

ASYMPTOTIC RELATIONS BETWEEN L - AND M -ESTIMATORS IN THE LINEAR MODEL

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Abstract. We obtain Bahadur-type representations for one-step L -estimators, M - and one-step M -estimators in the linear model. The order of the remainder terms in these representations depends on the smoothness of the weight function for L -estimators and on the smoothness of the ψ -function for M - and one-step M -estimators. We use the representations to investigate the asymptotic relations between these estimators. In particular, we show that asymptotically equivalent L - and M -estimators of the slope parameter exist even when the underlying distribution is asymmetric. It is important to consider the asymmetric case for both practical and robustness reasons: first, there is no compelling argument which precludes asymmetric distributions from arising in practice, and, secondly, even if a symmetric model can be posited, it is important to allow for the possibility of mild (and therefore difficult to detect) departures from the symmetric model.

Key words and phrases: Bahadur representations, multiple regression, robust estimators, uniform asymptotic linearity.