

## ASYMPTOTIC RELATIONS BETWEEN $L$ - AND $M$ -ESTIMATORS IN THE LINEAR MODEL

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**Abstract.** We obtain Bahadur-type representations for one-step  $L$ -estimators,  $M$ - and one-step  $M$ -estimators in the linear model. The order of the remainder terms in these representations depends on the smoothness of the weight function for  $L$ -estimators and on the smoothness of the  $\psi$ -function for  $M$ - and one-step  $M$ -estimators. We use the representations to investigate the asymptotic relations between these estimators. In particular, we show that asymptotically equivalent  $L$ - and  $M$ -estimators of the slope parameter exist even when the underlying distribution is asymmetric. It is important to consider the asymmetric case for both practical and robustness reasons: first, there is no compelling argument which precludes asymmetric distributions from arising in practice, and, secondly, even if a symmetric model can be posited, it is important to allow for the possibility of mild (and therefore difficult to detect) departures from the symmetric model.

*Key words and phrases:* Bahadur representations, multiple regression, robust estimators, uniform asymptotic linearity.